A Guide to Using This Text

What follows is a suggested schedule of the chapters and sections to be covered (and not covered) in a “typical” introductory course on differential equations, along with some commentary on the material. This discussion is directed towards the instructors teaching such a course. Keep in mind that these are merely suggestions. Each instructor should use their own good judgement and adjust this schedule as appropriate so that their course better suits the backgrounds of their students, the reasons they are taking the course, the time available, and the instructor’s own views on how the course should be taught.

Part I: The Basics

1. The Starting Point

Cover all of sections 1.1 and 1.2.

Section 1.3 should be covered quickly, with the understanding that your students’ understanding of and respect for this material will develop as they learn more about differential equations.

2. Integration and Differential Equations

Cover sections 2.1 and 2.2 fairly quickly, emphasizing that this is stuff the students should have already seen and be familiar with. Let them know that much of homework is a review of the basic integration methods they will be using extensively for the rest of the course. Do not, however, skip these sections or skip on the homework. Many of your students will probably need the review.

It seems that the material in sections 2.3 and 2.4 (on using definite integrals) is rarely part of introductory differential equation courses. It can be skipped. Still, it would not hurt to mention that using definite integrals makes it much easier to numerically solve those directly-integrable differential equations like

$$\frac{dy}{dx} = e^{-x^2} \quad ,$$

which require integrating integrals that are not easily integrated.

Part II: First-Order Equations

3. Some Basics about First-Order Equations

Sections 3.1 and 3.2 are fundamental and should not be skipped. Section 3.3 (on existence and uniqueness) should only be briefly discussed, and that discussion should probably be limited to theorem 3.1. Most instructors will want to skip the rest of chapter 3. (Still, you might tell the more mathematically inquisitive that the Picard iteration method developed in section 3.4 is pretty cool, and that the discussion is fairly easy to follow since the boring parts have been removed and stuck in the sections 3.5 and 3.6.)

4. Separable First-Order Equations

Cover sections 4.1, 4.2, 4.3 and 4.4.

You can skip sections 4.5 (*Existence, Uniqueness and False Solutions*), 4.6 (*On the Nature of Solutions to DEs*) and 4.7 (*Using and Graphing Implicit Solutions*). In an idea world, the material in these sections would be recognized as important for understanding "solutions". But this is not the ideal world and there isn't enough time to cover everything. Tell your students that they can read it on their own, and that understanding this material will help lead them to enlightenment.

Also skip section 4.8. It's on using definite integrals.

5. Linear First-Order Equations

Cover sections 5.1 and 5.2. Sections 5.3 and 5.4 can be ignored by most.

6. Simplifying Through Substitution

Cover the whole thing. Students should be able to recognize and use the "obvious" substitutions in sections 6.2 and 6.3. They should also be able to use any other reasonable substitution suggested to them. On the other hand, I see little value in memorizing the substitution for the Bernoulli equations — being able to derive it (exercise 6.5), however, is of great value.

7. Exact Equations and General Integrating Factors

Whether or not this chapter is covered depends on the background of your students. If they have had a course covering calculus of several variables — in particular, if they are acquainted with the multidimensional chain rule — then all of this chapter should be covered. After all, the methods for dealing with first-order differential equations discussed in the previous chapters are all just special cases of the general methods discussed in this chapter.

On the other hand, many introductory courses in differential equations do not have multivariable calculus as a prerequisite, and the students in those courses can not be expected to know the multidimensional chain rule on which almost everything in this chapter is based. That makes covering this material problematic. For that reason, this chapter and the rest of the text was written so that this chapter could be omitted. (But students in these courses should realize that they will want to read this chapter on their own after learning about the multidimensional chain rule.)

8. Slope Fields: Graphing Solutions Without the Solutions

Cover sections 8.1 and 8.2. These sections describe what slope fields are, why they can be useful, and how to construct and use them. I've also tried to make the exercises more meaningful that just "sketch a bunch of solution curves". If you want a computer assignment (highly recommended), require exercise 8.8.

Sections 8.3, 8.4 and 8.5 cover useful stuff regarding slope fields that is rarely discussed in introductory differential equation courses. If time permits, a *brief* discussion of stability as approached in section 8.3 may be enlightening, as might a discussion of "problem points", as done in section 8.4, just to illustrate how solutions can go bad.

9. Euler's Numerical Method

This development of Euler's numerical method for first-order differential equations follows naturally from the discussion of slope fields in the previous chapter. Cover sections 9.1 and 9.2, and then briefly comment on the material in sections 9.3 and 9.4 regarding the errors that can arise in using Euler's method. The detailed error analysis in section 9.5 is only for the most dedicated.

(By the way, you can go straight from chapter 8 to chapter 10, and cover chapter 9 later. If a chapter must be sacrificed because of time limitations, this would probably be one of the better candidates for the sacrifice.)

10. The Art and Science of Modeling with First-Order Equations

Cover sections 10.1 through 10.6. Consider section 10.7 on thermodynamics (Newton's law of heating and cooling) as optional. Section 10.8 is extremely optional; it covers technical issues that only mathematicians worry about.

Add few applications of your own if you want.

By the way, my approach to "applications" is a bit different from that found in many other texts. I prefer working on the reader's ability to derive and use the differential equations modeling any given situation, rather than flashing big list of applications.

Part III: Second- and Higher-Order Differential Equations

11. Higher-Order Equations: Extending First-Order Concepts

Cover section 11.1. That material is needed for future development. I'll leave it to you to decide whether section 11.2 is worth covering; it can be safely skipped.

Cover section 11.3 on initial-value problems.

Section 11.4 is on the existence and uniqueness of solutions to higher-order differential equations, and should only be briefly discussed, if at all. At most, mention that the existence and uniqueness results for first-order equations have higher-order analogs.

12. Higher-Order Linear Equations: Introduction and Basic Theory

Cover sections 12.1, 12.2 and 12.3. That material is fundamental.

Section 12.4 can help the students better understand "differential operators" in the context of differential equations, but the material in this section is only used later in the text to prove a few theorems. Since you are not likely to be discussing these particular proofs, you can safely skip this section. Suggest this section as enrichment for your more inquisitive students.

13. Reduction of Order

Cover sections 13.1 and 13.2. They develop the standard reduction of order method. This material is used later.

Extensions of this method are discussed in sections 13.3 and 13.4, along with explanations as to why these extensions are rarely used in practice. Don't cover these sections, but, to hint at future developments, mention that the variation of parameters method for solving nonhomogeneous equations, which will be developed later, is just an improvement on the reduction of order method for nonhomogeneous equations discussed in section 13.3.

14. Homogeneous Linear Equations — The Big Theorems

Cover sections 14.1, 14.2 and 14.3.

Section 14.4 concerns Wronskians. Consider it optional. This section was written assuming the students had *not* yet taken a course in linear algebra. Under this assumption, I do not feel that Wronskians are worth any more than a brief mention here. It's later that Wronskians become truly of interest.

15. Homogeneous Linear Equations — Verifying the Big Theorems

Completely skip this chapter. It's there just to assure readers that I didn't make up the big theorems in the previous chapter. Of course, you can read it for your own personal enjoyment and so that you can tell me of all the typos in this chapter.

16. Second-Order Homogeneous Linear Equations with Constant Coefficients

This may be the most important chapter for many of your students. By the end of the term, they should be able to solve these equations in their sleep. I also consider the introduction of the complex exponential as a practical tool for doing trigonometric function computations to be important.

Cover everything in this chapter except, possibly, example 16.5 on page 351 (that example concerns using the complex exponential to derive trigonometric identities — a really nice thing, but you'll probably be a little pressed for time and will want to concentrate on solving differential equations.)

17. Springs: Part I

This is a fairly standard discussion of unforced springs (maybe with more emphasis than usual on the modeling process). Cover all of it quickly.

18. Arbitrary Homogeneous Linear Equations with Constant Coefficients

Cover all of sections 18.1, 18.2 and 18.3.

Don't cover sections 18.4 and 18.5. They contain rigorous proofs of theorems naively justified earlier in the chapter. Besides, you will need the material from section 12.4 (which you probably skipped).

19. Euler Equations

Cover sections 19.1 and 19.2. Section 19.3 is optional. Don't bother covering section 19.4, though you may want to briefly comment on that material (which relates Euler equations to equations with constant coefficients via a substitution).

By the way, the approach to Euler equations taken here is taken to help reinforce the students' grasp of the theory developed for general linear equations. It will also help prepare them for the series methods (especially the method of Frobenius) that some of them will later see. That is why I downplay the substitution method commonly used by others.

20. Nonhomogeneous Equations in General

Cover sections 20.1 and 20.2. Briefly mention that reduction of order can be used to solve nonhomogeneous equations (that's really all that's done in section 20.3).

21. Method of Undetermined Coefficients

Cover everything in this chapter except section 21.7.

If you are running short on time, you might incorporate a brief discussion of resonance with the development of the material in section 21.3, and then skip the next chapter.

22. Springs: Part II

This is the material on forced oscillations in mass/spring systems. If you have time, cover sections 22.1, 22.1 and 22.3. Consider 22.4 as very optional.

23. Variation of Parameters

Cover sections 23.1 and 23.2. Do not cover section 23.3 unless you really feel that the variation of parameters formula is worth memorizing (I don't).

Part IV: The Laplace Transform

24. The Laplace Transform (Intro)

Cover sections 24.1, 24.2, 24.3, 24.4 and 24.5. (Don't waste much time on the gamma function — just about everything else in this chapter is much more important).

The students should at least skim the first part of section 24.6. This discusses some issues regarding piecewise continuous functions. In this text, I've adopted the view that "the value of a function at a point of discontinuity is 'irrelevant' ". Basically, I'm introducing the idea of functions being equal "almost everywhere" in the sense normally encountered in more advanced analysis courses. This is a practical and philosophical decision explained in the first part of section 24.6. Point this out in class, but don't make a big deal of it.

25. Differentiation and the Laplace Transform

Cover the material in sections 25.1 and 25.2

Section 25.3 is optional and concerns the integration identities for the Laplace transform. If you have time, do it; otherwise, just mention the existence of such identities as a counterpoint to the differentiation identities.

Section 25.4, which is mainly concerned with a rigorous derivation of the "derivative of a transform" identity, is very optional, though some may be intrigued by example 25.6 on page 519, which illustrates that, sometimes, that which seems obviously ture is not always true.

26. The Inverse Laplace Transform

Cover all of this chapter. And, yes, I really do wait until here to introduce the inverse transform. Trust me, it works.

27. Convolution

At least cover sections 27.1 and 27.2. If time allows, cover 27.3 on Duhamel's principle — this is an introduction to a very important concept in a wide range of applications (including ordinary and partial differential equations, and the generic modeling of systems in physics, optics and engineering). Ultimately, it more justifies the development of convolution than does the formula $\mathcal{L}^{-1}[FG] = f * g$.

28. Piecewise-Defined Functions and Periodic Functions

At least cover the material in sections 28.1 and 28.2. The material in section 28.3 is the natural extension of that in the previous sections — at least discuss the rectangle function since it is used in the next chapter. Cover the rest of section 28.3 if you have time. You might also suggest that your students at least skim section 28.4 if they suspect that they might ever have to compute convolutions with piecewise-defined functions.

Section 28.5 is on periodic functions. Consider it optional.

Section 28.6 is just a table of identities for the Laplace transform.

Section 28.7 generalizes the discussion of resonance from chapter 22. It is a nice application of Duhamel's principle which you certainly will not have time to cover. Maybe you will want to read it for yourself, or recommend it as enrichment for an interested student.

29. The Delta Function

Cover sections 29.1, 29.2 (at least the material on "strong brief forces") and 29.3. If you covered Duhamel's principle, you will want to discuss section 29.4. Don't bother with section 29.5, other than to recommend it to the interested students.