1

Intro

1.1 Some of What We Will Cover

Here is a very rough idea of what the first part of the "Math Physics" course will cover:

1. Linear Algebra
   (a) We will start with a fundamental development of the theory for "traditional" vectors (developed in a manner that, hopefully, will give you a better understanding of "tensors", later),
   (b) and then go into the theory for general vector spaces (with real and complex scalars). (This underlies much of the mathematics in physics.)
   (c) There will be much discussion on changing bases, and on particular types of linear operators/matrices (especially those that are orthogonal, unitary or Hermitian). (By the way, this will tie in with our future discussion of partial differential equations.)
2. Calculus in Space/Field Theory/Elementary Tensor Analysis
   (a) This begins with a discussion of position in space, curves and coordinate systems in Euclidean and non-Euclidean spaces.
   (b) We will discuss classical field theory (gradients through the divergence and Stokes theorems).
   (c) Finally, we will attempt to generalize our notions.

You should have already seen some of the material we will cover. A major goal is for you to gain a deeper "gut-level" understanding of the basic material, in addition to just being able to crank through computations.

1.2 Some Basic Mathematical Entities in Physics

The following will either be used extensively or studied extensively in the next few weeks:

Scalars: Scalars are numbers — integers, real numbers, complex numbers, … . We will start by using real numbers as scalars, but will quickly proceed to using complex numbers as scalars.

Here is some relevant notation:

$$\mathbb{N} = \{1,\ 2,\ 3,\ \ldots\} = \text{the set of natural numbers}$$

$$\mathbb{Z} = \{\ldots,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ \ldots\} = \text{the set of all integers}$$

$$\mathbb{R} = (-\infty, \infty) = \text{the set of all real numbers}$$

$$\mathbb{C} = \text{the set of all complex numbers}$$

Number/Scalar Arrays: These are simply sets of numbers arranged in some one- or two- (or higher-) dimensional arrangement. For example:

$$(1, 3, 5) \quad \text{or} \quad \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & 3 \\ 5 & 8 \\ -7 & 3 \end{bmatrix} \quad \text{or} \quad \cdots \quad .$$

Here is some relevant notation (n denotes an arbitrary positive integer):

$$\mathbb{R}^2 = \{(x, y) : x, y \in \mathbb{R}\} = \text{the set of ordered pairs of real numbers}$$

$$\mathbb{C}^2 = \{(z, \zeta) : z, \zeta \in \mathbb{C}\} = \text{the set of ordered pairs of complex numbers}$$

$$\begin{aligned} \mathbb{R}^n &= \{(x_1, x_2, \ldots, x_n) : x_k \in \mathbb{R} \text{ for } k = 1, 2, \ldots, n\} \\ &= \text{set of ordered } n\text{-tuples of real numbers} \end{aligned}$$

$$\begin{aligned} \mathbb{C}^n &= \{(z_1, z_2, \ldots, z_n) : z_k \in \mathbb{C} \text{ for } k = 1, 2, \ldots, n\} \\ &= \text{set of ordered } n\text{-tuples of complex numbers} \end{aligned}$$

Vectors: Traditionally, these are "arrows in space". In fact, we will end up with at least three *nonequivalent* meanings for the word "vector".

Points in Space: These are points/positions/locations in some "region in space". Take the meaning of "region in space" very loosely. It may be that this space consists of all positions in the universe or the set of points on some idealized sphere (the *surface* of some ball) or the surface of some warped sheet or something more abstract. When we have such a space, we will assume the different points can be connected by curves, and that we have a reasonable notion of the distance along a curve between two points, as well as a reasonable notion of the angle at which two intersecting curves cross. We may even assume more. In particular, if the set of points is a "Euclidean space", we will assume that everything they taught us in high school geometry is true.

It will be important to realize that "number arrays", "vectors" and "positions" are intrinsically *different* things. We often do use number arrays such as $(1, 2, 4)$ to *describe* vectors and positions. We do this by imposing a vector basis (for a vector space) or a coordinate system (for a position space), but these are artificial constructions. The physics — and hence, to some extent, the fundamental mathematics — must *not* depend on which vector basis or coordinate system we happen to chose.

Also keep in mind that, while vectors can be used to describe position (in Euclidean spaces, at least), vectors and position are two basically different things.

1.3 Describing Mathematical/Physical Objects and Phenomenon

As we go through the linear algebra and multidimensional calculus part of this course, really try to distinguish between the following three ways of describing mathematical entities and physical phenomenon (e.g., vectors, moving objects, electric field strengths, etc.):

Basis/Coordinate Free Description: Here, we make *no* use of coordinates or bases or components.

This is the way God does physics.

Basis/Coordinate Independent Description: Here, coordinate systems and/or bases are used, *BUT* the formulas based on the coordinate systems/bases remain essentially the same no matter which coordinate system or basis we chose. (We may actually restrict ourselves to "orthonormal" vector bases or "orthonormal" coordinate systems.)

This is the way Einstein did physics.

Basis/Coordinate Dependent Description: Here, the formulas used depend strongly on the choice of the vector basis and/or coordinate system.

Not a good way to describe very general phenomenon, but a cleverly chosen basis or coordinate system can often simplify computations.