Homework Handout VI

A. Find all separable solutions to each of the following pdes. Assume c denotes "some positive constant".

1. $\frac{\partial u}{\partial t} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$

2. $\frac{\partial u}{\partial t} + c^2 \frac{\partial^2 u}{\partial x^2} = 0$

3. $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$

4. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

B. "Separate" each of the following into two odes, and solve the odes that you can solve.

1. $i\beta \frac{\partial u}{\partial t} + \alpha^2 \frac{\partial^2 u}{\partial x^2} = Vu$ where α, β, and V are positive constants

2. $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$ (This is $\nabla^2 u = 0$ in polar coordinates.)

C. For each of the following, assume

$$u(\boldsymbol{x}, t) = g(\boldsymbol{x})\, h(t) \qquad \text{with} \qquad \boldsymbol{x} = (x_1, x_2, \dots)$$

and separate the pde into an ode for $h(t)$ and a pde for $g(\boldsymbol{x})$. Also, verify for yourself that, at least for some values of the separation constant, the pde for $g(\boldsymbol{x})$ is the Helmhotz equation (equation (9.29) on page 415 of AW&H). For each equation c denotes some positive constant.

1. $\frac{\partial u}{\partial t} - c^2 \nabla^2 u = 0$

2. $\frac{\partial^2 u}{\partial t^2} - c^2 \nabla^2 u = 0$