MA/PH 609
Howell
01/31/2014

Homework Handout III

In the following, x and y are real variables while z is a complex varible with $z = x + iy$.

A. Find the real and imaginary parts of $\frac{1}{2+3i}$ and $(2+3i)^2$.

B. Find the real part, $u(x,y)$, and the imaginary part, $v(x,y)$, for each of the following choices of $f(z)$:

$$z^2 \quad , \quad z^3 - 4 \quad , \quad \frac{1}{z} \quad , \quad e^z \quad , \quad z^* \quad , \quad (z^*)^2 \quad , \quad \sin(z)$$

C. Using the complex exponential, verify each of the following trig. identities:

1. $\sin(A)\sin(B) = \frac{1}{2}\Big[\cos(A-B) - \cos(A+B)\Big]$

2. $\sin(\theta)\cos(\theta) = \frac{1}{2}\sin(2\theta)$

3. $\sin^2(\theta) = \frac{1}{2}\Big[1 - \cos(2\theta)\Big]$

D. Find the relations between the hyperbolic trig. functions $\cosh(z)$ and $\sinh(z)$, and normal trig. functions $\cos(iz)$ and $\sin(iz)$. (Use the exponential formulas for $\cosh(z)$ and $\sinh(z)$, which are just as you should recall for $\cosh(x)$ and $\sinh(x)$ with the real variable x replaced by the complex variable z . If you don't recall the formulas for the hyperbolic cosine and hyperbolic sine, look them up!)

E. Skim §1.8 of AW&H and on page 61, do: 6, (7),

F. Which of the following is/are analytic on $\mathbb{C}$?

$$f(x+iy) = \left(x^2 - y^2\right) + i(2xy) \quad , \quad f(x+iy) = \left(x^2 + y^2\right) + i(2xy)$$

G. § 6.2 of AW&H (read it if you want), page 476: (2), 3, (10)

H. Evaluate the following integrals, and then look at problems 11.3.3 and 11.3.4 on page 485 of AW&H.

1. $\displaystyle\int_{3+4i}^{4-3i} \left(4z^2 - 3iz\right) dz$ ***2.*** $\displaystyle\int_{\pi(1+i)}^{i\pi} \cos(2z)\, dz$