Homework Handout III

A. Read § 4.1 in the online notes, doing all in-notes exercises.
Also skim/read §2.2 of AWH, *skipping* **Derivatives of Determinants** , **Systems of Linear Equations** , and **Determinant Product Theorem** on pages102 and 103, but read about the stuff on "direct products" and "trace" even though we won't do much with them). Also skip **Functions of Matrices** on page 113.
Starting on page 114 of AWH, "be sure you can do" # 1, 2 (note: $[\mathbf{A}, \mathbf{B}] = \mathbf{AB} - \mathbf{BA}$ is the "commutator" for **AB**), 7, (9), (11 'Pauli spin matrices'), (12), 21, 22, 23 (24 may also be interesting — look at it, at least).

B. Compute $\mathbf{A}^2$ where $\mathbf{A} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$, and then use your result to find a nonzero 2×2 matrix $\mathbf{A}$ satisfying $\mathbf{A}^2 = \mathbf{0}$.

C. Find the conjugate, the transpose, and the adjoint for each of the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1i & 2i \\ 3i & i \end{bmatrix} \quad , \quad \mathbf{B} = \begin{bmatrix} 3 & 2+3i & 5i \\ -2i & 8 & 3-4i \end{bmatrix} \quad , \quad \mathbf{C} = \begin{bmatrix} 2+3i \\ 4-5i \\ 7+8i \end{bmatrix}$$

D. What should a and b be so that $\begin{bmatrix} 2 & 3+5i \\ a & b \end{bmatrix}$

1. is symmetric? ***2.*** is Hermitian (i.e., self adjoint)?

E. Give an example of a 2×2 anti-Hermitian matrix with no zero entries.

F. Starting on page 114 of AWH, do 33, 34, 35.

G. Let $|\,\mathbf{a}\,\rangle = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $|\,\mathbf{v}\,\rangle = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ (assume the basis is orthonormal). Compute the following (remember, scalars are complex) and, if possible, compare each with $\langle\,\mathbf{a}\,|\,\mathbf{v}\,\rangle$:

1. $\langle\,\mathbf{a}\,|\otimes|\,\mathbf{v}\,\rangle$ ***2.*** $|\,\mathbf{v}\,\rangle\otimes\langle\,\mathbf{a}\,|$ ***3.*** trace$(\langle\,\mathbf{a}\,|\otimes|\,\mathbf{v}\,\rangle)$

H. Read § 4.3 and § 4.4 in the **Elementary Matrix Theory** notes, doing any in-text or in-lecture exercises. Also skim § 3.1 of AWH.
Starting on page 93 of AWH, do # 1 and 2 (rewrite the system in "matrix/vector form" first, and then think about what the problem is really about). Then do #5b on page 115.