

Homework Handout XI

A. For each of the following coordinate systems:

- i.* Pick a few points and, at each of the points, sketch the basis of tangents $\{\vec{\epsilon}_1, \vec{\epsilon}_2, \dots\}$ and the corresponding reciprocal basis $\{\vec{\epsilon}^1, \vec{\epsilon}^2, \dots\}$, and
- ii.* Find the reciprocal basis fields in terms of the given coordinate system, and the \mathbf{i} , \mathbf{j} and, if appropriate, \mathbf{k} vectors. Do this by first recalling (or re-deriving) the $\vec{\epsilon}_i$'s obtained in previous exercises, and then find the $\vec{\epsilon}^j$'s by solving $\vec{\epsilon}_i \cdot \vec{\epsilon}^j = \delta_i^j$ for each pair (i, j) . You can also use the result of the exercise in the notes about the reciprocal basis field when the coordinate system is orthogonal.

1. Polar coordinates
2. Spherical coordinates
3. The first coordinate system on the sheet *Three Coordinate Systems for the Plane*.
4. The second coordinate system on the sheet *Three Coordinate Systems for the Plane*.
5. The parabolic coordinate system from problem **L** in *Homework Handout VII*.

B. Find the g_{ij} 's and g^{ij} 's for each of the following coordinate systems:

1. Polar coordinates
2. Spherical coordinates
3. The first coordinate system on the sheet *Three Coordinate Systems for the Plane*.
4. The second coordinate system on the sheet *Three Coordinate Systems for the Plane*.
5. The parabolic coordinate system from problem **L** in *Homework Handout VII*.

C. Let \mathbf{F} and \mathbf{G} be the vector fields given by

$$\mathbf{F}(u, v) = u^2 \vec{\epsilon}_u + (u + 3v) \vec{\epsilon}_v \quad \text{and} \quad \mathbf{G}(u, v) = 7 \vec{\epsilon}^u + 8 \vec{\epsilon}^v$$

Find the co- and contravariant components of each assuming $\{(u, v)\}$ is

1. the polar coordinate system $\{(\rho, \phi)\}$
2. the first coordinate system on the sheet *Three Coordinate Systems for the Plane*.
3. the second coordinate system on the sheet *Three Coordinate Systems for the Plane*.
4. the parabolic coordinate system from problem **L** in *Homework Handout VII*.