Homework Handout I

Note: In skimming/reading the indicated sections of AWH (Arfken, Weber and Harris) keep in mind that their basic approach to the traditional vector theory is quite different from your instructor's. They typically depend on coordinate formulas to define such things as the dot product; we don't. Consequently, some of the issues they are concerned with (such as the invariance of the norm under rotations) are non-issues for us. Also, we will develop a much better approach to "rotations" when we get to the more general development of linear algebra.

In each of the following problems, first ask yourself if the problem can be done "component free". If not, go ahead and use components (even though we may not have yet properly developed them) — they were, after all, invented for computations, and you should already have a naive understanding of them.

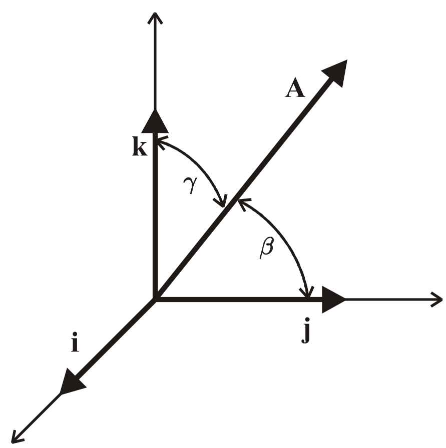
A. Skim through §1.7 and §3.1 of AWH; starting on page 52, do: 1, 2*, 4, 5.
(* about 2: Think geometrically, and you will realize that there is really nothing "to show"!)

B. On page 52, do 6 (get component formulas for these things), 7, 8.

C. Using the linearity of the dot product, show that $(\mathbf{v}+\mathbf{w})\cdot(\mathbf{v}-\mathbf{w}) = \|\mathbf{v}\|^2 - \|\mathbf{w}\|^2$.

D. The following assumes a traditional 3-dimensional vector space. Let $\{\mathbf{i},\mathbf{j},\mathbf{k}\}$ be a standard basis (i.e., any right-handed, orthonormal basis).

1. (The Cones Problem) Let $\mathbf{A}$ be a vector (as sketched in the figure) and let β and γ be, respectively, the angles between $\mathbf{A}$ and the basis vectors $\mathbf{j}$ and $\mathbf{k}$.



a. Find the components of $\mathbf{A}$ in the directions of $\mathbf{j}$ and $\mathbf{k}$.

b. Find the *two* possible components of $\mathbf{A}$ in the direction of $\mathbf{i}$. When is there only one such component?
(It may help to visualize β and γ as defining two cones about $\mathbf{j}$ and $\mathbf{k}$ — then, again, it may not.)

2. Previous editions of Arfken and Weber had the following problem:

The interaction energy V between two dipoles of moments $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ may be written in the vector form

$$V = -\frac{\boldsymbol{\mu}_1\cdot\boldsymbol{\mu}_2}{r^3} + \frac{3(\boldsymbol{\mu}_1\cdot\boldsymbol{r})(\boldsymbol{\mu}_2\cdot\boldsymbol{r})}{r^5}$$

and in scalar form

$$V = \frac{\mu_1 \mu_2}{r^3}(2\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos\varphi)$$

where $\mathbf{r}$ *is the vector from the first dipole to the second,*

$$r = \|\boldsymbol{r}\| \quad , \quad \mu_i = \|\boldsymbol{\mu}_i\| \quad , \quad \theta_i = \theta(\boldsymbol{r}, \boldsymbol{\mu}_i)$$

and φ *is the azimuth of* $\boldsymbol{\mu}_2$ *realitive to the plane containing the vectors* $\mathbf{r}$ *and* $\boldsymbol{\mu}_1$ *.*

a. Compute the vector formula for V using $\mathbf{r} = \mathbf{j}$, $\boldsymbol{\mu}_1 = \mathbf{i} + \mathbf{j}$, and $\boldsymbol{\mu}_2 = \mathbf{j} + \mathbf{k}$. Does this agree with the scalar formula given for V in that problem?

b. Starting with the given vector formula for V, derive the *correct* scalar formula for V. (Hints: Redraw the vectors all starting at the same point with $\mathbf{r}$ in the direction of $\mathbf{j}$ and with $\boldsymbol{\mu}_1$ in the same plane as $\mathbf{i}$ and $\mathbf{j}$. You may then treat φ as either the angle between $\boldsymbol{\mu}_2$ and $\mathbf{k}$, or as the angle between $\boldsymbol{\mu}_2$ and the XY-plane — use whichever you prefer. Also, use results from the Cones Problem, above!)

E. Read the section on cross products in the Traditional Vectors virtual handout, doing all in-text exercises. Then, lightly skim the cross produce subsection of §3.2 in AWH; and starting on page 131, do: 1, 2, 4, 5, 7,

F. For this problem, assume the following four points have been plotted on an ordinary XYZ coordinate system in Eucidean space:

$$\boldsymbol{O} = (0, 0, 0) \quad , \quad \boldsymbol{A} = (1, 2, 3) \quad , \quad \boldsymbol{B} = (3, 5, 7) \quad \text{and} \quad \boldsymbol{C} = (4, 2, 5) \quad .$$

As usual, let $\mathbf{i}$ be the vector from $\boldsymbol{O}$ to $(1, 0, 0)$; $\mathbf{j}$ the vector from $\boldsymbol{O}$ to $(0, 1, 0)$, and $\mathbf{k}$ the vector from $\boldsymbol{O}$ to $(0, 0, 1)$.

a. Find the vectors $\overrightarrow{\boldsymbol{AB}}$ and $\overrightarrow{\boldsymbol{AC}}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$.

b. Find the point $\boldsymbol{D} = (x, y, z)$ so that $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$ and $\boldsymbol{D}$ are the corners of a parallelogram with side $\boldsymbol{AB}$ being parallel to side $\boldsymbol{CD}$.

c. Find the area of the above parallelogram.

G. Read the section on triple products in the Traditional Vectors virtual handout, doing the in-text exercises. Then skim §3.2 in AWH; and starting on page 131, do 8 and 12.

H. Using the linearity of the cross product, show that $(\mathbf{v} + \mathbf{w}) \times (\mathbf{v} - \mathbf{w}) = 2\,\mathbf{w} \times \mathbf{v}$.