
Contents

VI Systems of Differential Equations (More Complete)

38 Addendum to Chapter 38: Higher-Order Systems	38–1
38.1 Higher-Order Systems	38–1
Additional Exercises	38–4
39 Critical Points, Direction Fields and Trajectories—ALT	39–1
39.1 Important Terminology and Convenient Notation	39–1
39.2 Constant (or Equilibrium) Solutions	39–5
39.3 Sketching Trajectories for Autonomous Systems	39–10
39.4 Critical Points, Stability and Long-Term Behavior	39–14
39.5 Applications	39–17
39.6 Existence and Uniqueness of Trajectories	39–24
39.7 Proving Theorem 39.2	39–25
Additional Exercises	39–29
41 Homogeneous Linear Systems and Their General Solutions	41–1
41.1 Basic Terminology and Notation	41–1
41.2 More Terminology and Some Basic Results	41–4
41.3 The Main Result on General Solutions to Linear Systems	41–12
41.4 Wronskians and Identifying Fundamental Sets	41–12
41.5 Fundamental Matrices	41–17
41.6 General Solutions for a Single N^{th} -order Linear Differential Equation	41–17
Additional Exercises	41–21
42 Homogeneous Constant Matrix Systems, Part I	42–1
42.1 Basics and Some Fundamental Solutions	42–1
42.2 A Short Review of Eigenvalues and Eigenvectors (with Applications)	42–3
42.3 Eigenpairs and Corresponding Solutions	42–13
42.4 Two-Dimensional Phase Portraits: Preliminaries	42–14
42.5 Two-Dimensional Phase Portraits from Real, Complete Eigen-Sets	42–16
Additional Exercises	42–24
43 Homogeneous Constant Matrix Systems, Part II	43–1
43.1 Solutions Corresponding to Complex Eigenvalues	43–1
43.2 Two-Dimensional Phase Portraits with Complex Eigenvalues	43–6
43.3 ‘Second Solutions’ When the Set of Eigenvectors is Incomplete	43–11

43.4	Two-Dimensional Phase Portraits with Incomplete Sets of Eigenvectors	43–14
43.5	Complete Sets of Solutions Corresponding to Incomplete Sets of Eigenvectors . . .	43–17
43.6	Appendix for Sections 43.1 and 43.2	43–24
	Additional Exercises	43–28
44	Miscellaneous Topics Involving Homogeneous Constant Matrix Systems	44–1
44.1	Shifted Constant Matrix Systems	44–1
44.2	Classifying Critical Points for 2×2 Systems	44–3
44.3	Phase Portraits for Imprecisely Known Systems	44–4
44.4	Phase Portraits for Large Constant Matrix Systems	44–6
44.5	Using Fundamental and Exponential Matrices	44–7
44.6	Using Laplace Transforms	44–16
44.7	Euler Systems	44–16
44.8	Using Similarity Transforms	44–21
	Additional Exercises	44–25
45	Nonhomogeneous Linear Systems	45–1
45.1	General Theory	45–1
45.2	Method of Undetermined Coefficients / Educated Guess	45–3
45.3	Reduction of Order/Variation of Parameters	45–9
45.4	Using Laplace Transforms	45–16
45.5	Using Similarity Transforms	45–17
	Additional Exercises	45–20
46	Nonlinear Systems: Phase Plane Analysis Using Linearizations	46–1
46.1	The Systems of Interest and a Little Review	46–1
46.2	Rewriting Systems Using Jacobian Matrices	46–3
46.3	Linearized Systems and Trajectories Near Critical Points	46–8
46.4	Sketching Trajectories for Nonlinear Systems	46–12
46.5	Application: Competing Species	46–15
46.6	Re-Analyzing a Particular Competing Species Model	46–15
46.7	General Analysis of the Competing Species Model	46–21
46.8	Application: The Damped Pendulum	46–25
	Additional Exercises	46–29
47	Coordinate Curves for Trajectories	47–1
47.1	Coordinate Equations for Trajectories	47–1
47.2	Trajectories as Level Curves	47–3
47.3	Application: Foxes in the Rabbit Ranch	47–5
47.4	Application: The Ideal Pendulum	47–10
	Additional Exercises	47–13
48	The Liapunov Method for Determining Stability (DRAFT)	48–1
48.1	The Liapunov Method, Naively Developed	48–1
48.2	The Liapunov Theory: Rigorous Definitions and Results	48–7
48.3	Finding Liapunov Functions	48–10
48.4	Appendix – Technical Issues	48–13
	Additional Exercises	48–13