Chapter 8: Slope Fields

8.2 a. The slopes of the slope lines are found by plugging the given values for $x$ and $y$ into the differential equation, $\frac{dy}{dx} = \frac{1}{2} (x^2 + y^2)$:

At $(x, y) = (0, 0)$ the slope of the slope line is $y' = \frac{1}{2} (0^2 + 0^2) = 0$.
At $(x, y) = (1, 0)$ the slope of the slope line is $y' = \frac{1}{2} (1^2 + 0^2) = \frac{1}{2}$.
At $(x, y) = (0, 1)$ the slope of the slope line is $y' = \frac{1}{2} (0^2 + 1^2) = \frac{1}{2}$.
At $(x, y) = (1, 1)$ the slope of the slope line is $y' = \frac{1}{2} (1^2 + 1^2) = 1$.

Plotting each slope line at its corresponding point then gives the slope field sketched to the right.

8.2 c. The slopes of the slope lines are found by plugging the given values for $x$ and $y$ into the differential equation, $\frac{dy}{dx} = \frac{y}{x}$:

At $(x, y) = (1, 1)$ the slope of the slope line is $y' = \frac{1}{1} = 1$.
At $(x, y) = (\frac{3}{2}, 1)$ the slope of the slope line is $y' = \frac{1}{\frac{3}{2}} = \frac{2}{3}$.
At $(x, y) = (1, \frac{3}{2})$ the slope of the slope line is $y' = \frac{\frac{3}{2}}{1} = \frac{3}{2}$.
At $(x, y) = (\frac{3}{2}, \frac{3}{2})$ the slope of the slope line is $y' = \frac{\frac{3}{2}}{\frac{3}{2}} = 1$.

Plotting each slope line at its corresponding point then gives the slope field sketched to the right.

8.2 e. The slopes of the slope lines are found by plugging the given values for $x$ and $y$ into the differential equation, $\frac{dy}{dx} = \frac{1}{2} (x - y)^2$:

At $(x, y) = (0, 0), (1, 1)$ and $(2, 2)$: Here $x - y = 0$.
Thus, the slope of the slope line is $y' = \frac{1}{2} (x - y)^2 = \frac{1}{2} 0^2 = 0$.
At \((x, y) = (0, 1), (1, 0), (1, 2)\) and \((2, 1)\): Here \((x - y)^2 = 1\).

Thus, the slope of the slope line is \(y' = \frac{1}{2}(x - y)^2 = \frac{1}{2} \cdot 1 = \frac{1}{2}\).

At \((x, y) = (0, 2)\) and \((2, 0)\): Here \((x - y)^2 = 2^2 = 4\).

Thus, the slope of the slope line is \(y' = \frac{1}{2}(x - y)^2 = \frac{1}{2} \cdot 4 = 2\).

Plotting each slope line at its corresponding point then gives the slope field sketched to the right.