

Preparing for the Third Exam

The test covers all the material in

- sections 1 through 4 of chapter 40,
- (sections 2, 3 and 4 of chapter 41)¹,
- and all of chapter 43 ,

along with the material from previous sections needed for that material (e.g., finding eigenvalues and eigenvectors, knowing about direction fields, etc.). Reviewing the assigned homework from from these chapters (Mon. 3/14 through Mon. 4/7) would be a good idea.

In particular:

- ◆ As usual, know the basic notation and terminology. I will not ask for definitions, but if, for example, I ask a question involving “critical points” or “stability”, I expect you to know what these things are well enough to answer the question.
- ◆ Given a linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ where \mathbf{A} has complex eigenvalues, be able to do all of the following:
 - Find the eigenvalues and eigenvectors. (Actually, I may just give you one of the eigen-pairs.)
 - Classify the critical point $(0, 0)$ as a *spiral point* or *center*, and determine whether the corresponding equilibrium solution is *asymptotically stable* , *stable but not asymptotically stable* or *unstable*.
 - Sketch a phase portrait of the system.
 - Write out the solution in terms of real-valued functions only.
- ◆ Given a nonlinear system $\mathbf{x}' = \mathbf{F}(\mathbf{x})$, be able to find the critical points, compute the Jacobian matrix, construct the appropriate linearized systems, and then solve and use the eigenvalues and eigenvectors of the appropriate linearized systems to sketch a rough phase portrait of the given nonlinear system.
- ◆ Don't be surprised if I give you a nonlinear system describing some sort of “competing species” model with the critical points already found and with the appropriate eigenvalues and eigenvectors given at each critical point. You will then have to draw a rough phase portrait of the trajectories, and tell me a little about what this phase portrait is telling you about the eventual outcome of the system.
- ◆ Relax. Get a good night's sleep the night before, and don't cram at the last minute.

¹There will not be questions explicitly from the material in chapter 41. That material was to prepare us for chapter 43.