Preparing for the Second Exam

The test covers all the material we've covered in class on systems of differential equations from chapters 36 through 39 of the lecture notes. Reviewing the assigned homework from Monday 2/17 through Wednesday 3/12 would be a good idea. Do not plan on using any calculator or computer (or other electronic do-dad) during this test.

In particular:

- Know the basic notation and terminology. I will not ask for definitions, but if, for example, I ask a question involving "critical points" or "equilibrium solutions", I expect you to know what these things are well enough to answer the question. I will also expect you to be able to convert a system written as a "list of differential equations" to matrix/vector form and back.

- I may ask you to determine if a given set of functions is a solution to a given system of differential equations or to an initial-value problem involving a system.

- Be able to solve a relatively simple, weakly coupled system of differential equations.

- Expect to be given a single high-order differential equation to convert into system of first-order differential equations.

- Be able to find the constant/equilibrium solutions for a system (equivalently, the critical points for a system).

- Understand trajectories, direction fields and phase portraits. Expect to construct a "small" direction field (one or two points), and to have to sketch certain trajectories using a given direction field. Also, given a direction field, be able to tell me a little about the solutions to the corresponding system of differential equations as $t \to \pm\infty$.

- Be able to determine whether a given set of vector-valued functions is a fundamental set of solutions for a given system of differential equations. This includes the intelligent use of the Wronskian.

- Given a fundamental set of solutions for a homogeneous linear system, be able to solve initial-value problems involving that system. (Of course, you may have to find that fundamental set, if it is a sufficiently simple system.)

- Expect **multiple problems** involving homogeneous constant matrix systems of differential equations. I will limit the problems to systems whose matrices have complete sets of eigenvectors and only real eigenvalues (as in chapter 39). In particular:
  - Plan on solving one such system, finding the eigenvalues and eigenvectors yourself.

- Expect to be given one or more 2×2 constant matrix systems $\mathbf{x}' = \mathbf{A}\mathbf{x}$, along with the eigenvalues and eigenvectors for $\mathbf{A}$, and told to do some or all of the following:
  - Tell me if the equilibrium solution $\mathbf{x}(t) \equiv \mathbf{0}$ is *stable*, *asymptotically stable* or *unstable*.
  - Classify the critical point $(0, 0)$ as a *node* or *saddle point*.
  - Sketch a phase portrait of the system.

- Do not expect questions on "applications" on this test.