Preparing for the Final

The final is comprehensive. It will be worth 200 points, and will, essentially, be like taking two tests. There will be roughly

60 to 75 points on power series and modified power series solutions,

95 to 110 points on linear and nonlinear systems, and

25 to 35 points on boundary-value and Sturm-Liouville problems.

The following list pretty well tells you what you should know for this final. Obviously, I will not be able to test you on everything below, but expect the final to cover a fair representation of what's on this list.

General

- Know the basic notation and terminology. I will not ask for definitions, but if, for example, I ask a question involving "critical points" or "equilibrium solutions", I expect you to know what these things are well enough to answer the question.

- Know the material from *A Brief Review of Elementary Ordinary Differential Equations*, as well as the material reviewing series and power series (chapter 30 of the lecture notes) but do not expect any problems specifically on that material.

Power Series Solutions

- Be able to find all the singular points for any differential equation, and be able to determine which of those are regular singular points. Know how to use these points to find the largest open interval(s) on which the power series solution (or modified power series solution) about some point x_0 is guaranteed to converge.

- Be able to find the first few terms of the general power series solution about an ordinary point to some first- or second-order differential equation using the algebraic method. In your work, you will have to identify the recursion formula, and use it to compute several terms of the series (I will tell you how many). I will not ask you to identify the general formula for the terms.

- Plan on finding at least the first few terms of one "modified" power series solution to some differential equation about a regular singular point using the method of Frobenius. In your work, you may have to explicitly identify: the indicial equation, the corresponding exponents of the equation/singularity (i.e., the solutions r_1 and r_2 to the indicial equation), the r for which there is guarranteed to be a power series solution (the larger r), and the corresponding recursion formula. I will tell you how many terms of the series to compute. I will not ask you to identify the general formula for the terms.

- I may break up a "Frobenious problem" into pieces and just ask you to, say, find the indicial equation.

- Know what the big theorems on the Frobenius method (theorems 34.1 and 34.2) say about the indicial equation and its solutions, the existence of at least one series solution corresponding to "the larger r", when the method will yield a second power series solution, the general nature of the second solution when the method does not yield a second series (don't memorize the formulas for the second solutions, but do remember about the $\ln|x - x_0|$), and the convergence of the series.

- Be able to approximate the modified power series solutions described in theorem 34.2 by solutions to the corresponding Euler equation. In particular, be able to find

$$\lim_{x \to x_0} y(x)$$

for a given regular singular point x_0 and solution $y(x)$ corresponding to one of the exponents r_1 and r_2 (as in the exercises in set 34.2).

Systems of Differential Equations

- Be able to convert a given a single high-order differential equation to a system of first-order differential equations.

- Understand trajectories, direction fields and phase portraits. You may have to construct a "small" direction field (one to four points), and to have to sketch certain trajectories using a given direction field. Also, given a trajectory field, be able to tell me a little about the solutions to the corresponding system of differential equations as $t \to \pm\infty$.

- Be able to determine whether a given set of vector-valued functions is a fundamental set of solutions for a given system of differential equations. This includes the intelligent use of the Wronskian.

- Expect problems involving homogeneous constant matrix systems of differential equations. Be able to find the general solution to any such system of d.e.s that I throw at you. Expect at least one case where you have to find the eigenvalues and eigenvectors yourself, and a few cases in which I give you the eigenvalues and eigenvectors. On the final, all systems will be 2×2.

- Given any 2×2 constant matrix systems $\mathbf{x}' = \mathbf{A}\mathbf{x}$, along with the eigenvalues and eigenvectors for $\mathbf{A}$, be able to:

  Tell me if the equilibrium solution $\mathbf{x}(t) \equiv \mathbf{0}$ is a *stable*, *asymptotically stable* or *unstable* equilibrium solution.

  Classify the critical point $(0,0)$ as a *node*, *saddle point*, *spiral point* or *center*.

  Sketch a phase portrait of the system.

- Plan on finding the critical points for a nonlinear system, as well as the Jacobian for the system and the lineraized system about a point for a nonlinear system.

- I am, again, likely to give you a nonlinear system describing some sort of "competing species" model with the critical points already found and with the appropriate eigenvalues and eigenvectors given at each critical point. You will then have to draw a rough phase portrait of the trajectories, and tell me a little about what this phase portrait is telling you about the eventual outcome of the system.

Boundary-Value and Sturm-Liouville Problems

- Keep in mind that the material in chapter 46 of the notes was mainly to prepare you for chapter 47. You may have to solve one or more boundary-value problems, but that would only be part of finding all the solutions to some Sturm-Liouville problem.

- Expect to have to solve a simple Sturm-Liouville problem.

- Be able to convert a given 2nd order differential equation (with paramenter λ) to Sturm-Liouville form.

- The discussion on the last day of class was meant to indicate why this material is useful, and where we would be heading if we had the time. But no questions on the Final will come specifically from that discussion.