Preparing for the Final

The final is comprehensive. It will be worth 200 points, and will, essentially, be like taking two tests. The points will be "roughly evenly" divided between problems on

1. "preliminaries" and first-order differential equations,
2. higher-order linear differential equations, and
3. the Laplace transform,

You will be given the table of *Laplace Transforms of Common Functions (version 2)* and the table of *Commonly Used Identities (version 2)*. Copies of these are at the class web site.

The following list pretty well tells you what you should know for this final. Obviously, I will not be able to test you on everything described below, but do expect the final to cover a fair representation of what's on this list. Needless to say, if there is a topic below that you don't review because you don't think it will be on the exam, then it probably <u>will</u> be on the exam.

- Be able to <u>test</u> a given function to see if it is a solution to a given differential equation.

- Be able to identify first-order differential equations as being "*directly integrable*", "*separable*", "*linear*", or "*none of the above*". If the problem is to 'identify the d.e.', you will not be asked (and should not attempt) to solve the equation.

- Don't forget what constant/equilibrium solutions are.

- Be able to solve a separable first-order differential equation (it will be identitified as separable). You will have to find the general solution to it, and, possibly, also the particular solution satisfying some initial-value problem.

- Be able to solve a linear first-order differential equation (it will be identified as linear). You will have to find the general solution to it, and, possibly, also the particular solution satisfying some initial-value problem.

- Be able to solve a directly integrable differential equation but do not expect me to actually give you one, except, possibly, as something that arises after a substitution.

- Know how to use "substitution" in solving differential equations. I may ask you to just convert one d.e. to another using a substitution, or I may have you completely solve one. If the appropriate substitution is not obvously one of the three standard substitutions we discussed, then I will tell you the substitution to use.

- Be able construct a "small" slope field for a given differential equation, and be able to use a given slope field to graph (approximately) the solution to a given initial-value problem. (If I give you the slope field, I will not give you the actual d.e.)

- Be able to set up the differential equation modelling some given process (e.g.: A rabbit population over time), and then being able to answer simple questions about the process (e.g.: How many rabbits will we eventually have?). Do not waste your time memorizing equations already derived; I will not simply ask you to model something we've already done with just the numbers changed!

- I will ***not*** give you a second-order differential equations to be treated as first-order equations via the substitution $v = \frac{dy}{dx}$ *except*, possibly, as part of a "reduction of order" problem.

- Be able to find the general solution to a second-order, homogeneous differential equation using one given solution and the method of reduction of order.

- Be able to solve any homogeneous linear differential equation with constant coefficients that I throw at you. Expect at least one to have a characteristic polynomial with repeated roots and one with complex roots. All of your final answers will be required to be in terms of *real-valued* functions, even if the solutions to the characteristic equation are complex. Most of these equations will be second order, but expect one or two to be of even higher order.

- Be able to solve any second-order (homogeneous) Euler equation I throw at you. Again, all of your final answers will be required to be in terms of *real-valued* functions, even if the solutions to the indicial equation are complex.

- Be able to find both particular solutions and general solutions to nonhomogeneous equations with constant coefficients using the method of "undetermined coefficients" (also called the method of "educated guess"). I am likely to ask you to completely solve one nonhomogeneous equation, and to give me the "appropriate guess" (without determining the coefficients) for one to three other nonhomogeneous equations. Keep in mind that the "appropriate guess" might not be the "first guess".

- Be able to set up and solve the system of equations for u and v (or u_1 and u_2) in the variation of parameters method for solving nonhomogeneous equations. And, of course, know how to construct the general solution to the d.e. from these.

- Be able to solve initial-value problems, both those involving homogeneous differential equations and those involving nonhomogeneous differential equations.

(In solving the problems just described, be sure to demonstrate that you have a basic grasp of the basic theory we developed. Show that you know how to construct a general solution to a linear differential from an appropriate collection of particular solutions to either the given equation or, if necessary, the corresponding homogeneous equation, and that you know to ask things like "Do we have the right number of solutions?" and "What about 'linear independence'?")

- Be able to compute a Laplace transform using the basic integral definition.

- Be able to do the following problem:

  Using the basic definition, prove/derive any ***one*** *of the following two identities (your choice, but CIRCLE YOUR CHOICE!). You may assume* $F(s) = \mathcal{L}[f(t)]|_s$ *,* α *is a real number, and that the function(s) are all continuous, differentiable, "of exponential order", and everything vanishes at infinity.*

  Then will follow a list of just two of the following four identities:

  $$\mathcal{L}[\,e^{\alpha t} f(t)\,]|_s = F(s-\alpha)$$

  $$\mathcal{L}[\,f(t-\alpha)\,\text{step}(t-\alpha)\,]|_s = F(s)\,e^{-\alpha s} \qquad \text{for} \quad 0 \le \alpha$$

  $$\mathcal{L}[\,f'(t)\,]|_s = s\,F(s) - f(0)$$

  $$\mathcal{L}[\,t\,f(t)\,]|_s = -\frac{dF}{ds}$$

- Be able to compute several Laplace transforms and inverse Laplace transforms using the tables. Be good at recognizing which identities are appropriate and at using them!

- Be able to compute $\mathcal{L}^{-1}[F(s)\,G(s)]|_t$ using convolution and using (if appropriate) partial fractions. I may tell you which method to use.

- Be able to solve initial-value problems using the Laplace transform.

- Be able to handle piecewise-defined functions.

- Expect to see a delta function.